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LINEAR ARITHMETICAL FORMS OF K-VALUED FUNCTIONS AND THEIRS IMPLEMENTATION BY SYSTOLIC ARRAY

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Abstract - A new method to describe any multivalued logic functuon given by its truthtable, by means of a system of some so-called linear arithmetical polynomials (AP). An algorithm to represent some linear AP for given k-valued function systems and solution of reverse problem are proposed. A scope to realize algorithms on linear systolic arrays is demonstrated.

1.INTRODUCTION

The search of the most simple. convenient from organizing of computing the analytical expressions (forms) of multivalued logic functions (MVL-functions) is an actual problem in logic design. The traditional description forms for MVL-functions where the operations of algebra of logic are used. recently so-called AP and polynomial-similar forms were proposed. This allows to consider as result of one of directions of algebra of logic. i.e. arithmetical logic.The elements of arithmetical logic are used in artificial intelligence systems for a long time. Thus. in [1]there were considered the results on design the linear predicates for the set of masks for persceptron and synthesis of threshold ways to solve integer programming problems that. in ones turn. is based on the results of[2.3].In[4-8] arithmetical logic methods are used to solve some applied problems and in [9-12]the algebra of AP was developed and the digital signal processing methods were proposed to synthesize these forms. A method of synthesis of linear AP for a system of some Boolean functions is proposed in [13.14].The problem of synthesis of AP for k-valued functions is the less explored in the arithmetical logic [15] and also using the orthogonal transforms[11.12].

This paper presents a generalization of the method to synthesize linear AP for MVL-functions. The approach of the authors consists in that to represent a system of MVL-functions by linear AP by the way of decomposition and discrete orthogonal transform. The assumed system of some MVL-functions can be restored in an unique fashion.

2. INDEX TERMS

A k-valued logic function $f(X)=f(x_1,x_2,\ldots,x_n)$of n variables is the logic function given on the set {0.1.....k-1} Ordered sets of values of variables are all the possible k^n sets of variables $x_1,x_2,\ldots,x_n$ (arguments of MVL-function f(X)) from the set {0.1.....k-1} arranged in lecsigraphic order.The ordered sets of variables).when n=2. k=3 are 00. 01. 02. 10. 11. 12. 20. 21. 22. Basic set of values of variables $x_1,x_2,\ldots,x_n$ is such set whose values of x_j $(j=\overline{1,n})$ satisfy the condition: $x_j \le 1$ and $x_p=0\,(p=\overline{1,j-1};\overline{j+1,n})$.Their number is (n+1).For example. basic sets of variables when n=2 are 00. 01. 10.

Let a MVL-function is given by truthtable column vector $X = |x \;.\; x \;.\; \ldots \; x \quad|$ .where $x^{(i)}$ $(i=\overline{0,k^n-1})$ is the value of the function on the corresponding set.The elements of vector **X** with indexes $0,k^0,k^1,\ldots,k^{n-1}$ are values of the function on the basic sets of variables.If a system of m MVL-functions $f_0(X),f_1(X),\ldots,f_{m-1}(X)$ is given. then its truthtable column vector $\mathbf{X}_d$ is defined as weighted sum of vectors $\mathbf{X}_j$ $(j=\overline{0,m-1})$ of functions of the system

$$\mathbf{X}_d=\sum_{j=0}^{m-1}k^j\mathbf{X}_j, \tag{1}$$

The AP $P(X)=P(x_1,x_2,\ldots,x_n)$ of a k-valued logic function $f(X)=f(x_1,x_2,\ldots,x_n)$ of n variables is a multinomial of (k-1)-th power

$$P(X)=\sum_{i=0}^{k^n-1}p^{(i)}x_1^{i_1}x_2^{i_2}\ldots x_n^{i_n}; \qquad x_i^j=\begin{cases}1,\ j=0;\\ x_i^j,\ j\neq 0;\end{cases} \tag{2}$$

where i_j $(j=\overline{1,n})$ is the j-th digit of the k-valued representation of i: $p^{(i)}$ is the i-th coefficient of the AP (it is a real number).The AP of some MVL-functions are given in Table 1.

The values of a function f(X) and values of its AP are the same on the same sets of values of variables.Any MVL-function f(X) of n variable can be represented by AP in the form (2) [12.15].

Table 1. AP of some 3-valued logic functions

f(X)	AP
$\bar{x}_1$	$2-x_1$
$\hat{x}_1$	$1+(5/2)x_1-(3/2)x_1^2$
$x_1 \wedge x_2$	$(5/2)x_1x_2-x_1x_2^2-x_1^2x_2+(1/2)x_1^2x_2^2$
$x_1 \vee x_2$	$x_2+x_1-(5/2)x_1x_2+x_1x_2^2+x_1^2x_2-(1/2)x_2^2x_2^2$

Designate the coefficients column vector of AP (1) as $P=[p^{(0)},p^{(1)},\dots,p^{(k^n-1)}]$.Then the pair of orthogonal transforms $\mathrm{P}=K_{k^n}\mathrm{X}$ and $\mathrm{X}=K_{k^n}^{-1}\mathrm{P}$ (3) defines a signle connection between the truthtable column vector **X** of a MVL-function f(X) and the coefficients column vector **P** of its AP P(X). Here K_{k^n} and $K_{k^n}^{-1}$ are direct and inverse transform matrices: the $k^n \times k^n$ matrix K_{k^n} is defined by the rule $K_{k^n}=K_k \otimes K_{k^{n-1}}$, :$\otimes$ is denotes Kronecker product: the elementary matrix K_k is found from the equation (orthogonality condition) $K_k \times K_k^{-1}=I_k$,. where I_k is the $k \times k$ identity matrix: the elements $k_{i,j}^{-1}$ of the inverse matrix K_k^{-1} are defined by the rule $k_{i,j}^{-1}=i^j$. $(i,j=\overline{0,k-1})$. The matrices K_k and K_k^{-1} for some values of k are given in Table 2.

Table 2. Direct K_k and inverse K_k^{-1} orthogonal transform matrices

k	Direct transform matrix K_k	Inverse transform matrix K_k^{-1}
2	$\begin{bmatrix}1&0\\-1&1\end{bmatrix}$	$\begin{bmatrix}1&0\\1&1\end{bmatrix}$
3	$\frac{1}{2}\begin{bmatrix}2&0&0\\-3&4&-1\\1&-2&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\1&1&1\\1&2&4\end{bmatrix}$
4	$\frac{1}{6}\begin{bmatrix}6&0&0&0\\-11&18&-9&2\\6&-15&12&-3\\-1&3&-3&1\end{bmatrix}$	$\begin{bmatrix}1&0&0&0\\1&1&1&1\\1&2&4&8\\1&3&9&27\end{bmatrix}$

3.CONDITION TO REPRESENT A MVL-FUNCTION BY A LINEAR AP

Linear AP are the most convenient as for hardware and software realization. as to decrease computing complexity of linear AP. They are written as below:

$$P_l(X)=p_l^{(0)}+p_l^{(1)}x_n+p_l^{(2)}x_{n-1}+\dots+p_l^{(n)}x_1, \quad (4)$$

where $p_l^{(j)}$ $(j=\overline{1,n})$ are the coefficients of the AP. Linear AP (4) is AP (2) whose coefficients $p^{(i)}=0$ when $i=\overline{0,k^n-1}$ and $i \neq 0,k^0,k^1,\dots,k^{n-1}$.It cannot be represented any MVL-function by linear AP (4). We define the conditions. when it can do.

Theorem. A k-valued logic function f(X) of n variables may be represented by a linear AP P(X) if and only if the i-th element $(i=\overline{0,k^n-1})$ of the truthtable column vector **X** of this function meets the condition

$$x^{(i)}=\sum_{j=0}^{n-1}x^{(k^j)}i_{n-j}+x^{(0)}(1-\sum_{j=1}^{n}i_j), \quad (5)$$

where i_{n-j} is the (n-j)-th digit of k-valued representation of i.therefore. to represent a MVL-function f(X) by linear AP. every element of vector **X** ought to depend on values of the function on basic set of variables.

Find a matrix analogue of the condition. For this we write expression (5) as follows

$$x^{(i)}=x^{(k^{n-1})}i_1+x^{(k^{n-2})}i_2+\dots+x^{(k^0)}i_n+x^{(0)}(1-\sum_{j=1}^{n}i_j).$$

Form from the rows $[i_1\ i_2 \dots i_n(1-\sum_{j=1}^{n}i_j)]$ $k^n \times (n+1)$ matrix T taking into account that $i_j=\overline{0,k-1}$:

$$T=\begin{bmatrix}0&0&\dots&0&1\\0&0&\dots&1&0\\ \cdot&\cdot&\cdot&\cdot&\cdot\\k-1&k-1&\dots&k-1&1-n(k-1)\end{bmatrix}$$

Then the condition of existing the linear AP for a given MVL-function is written in matrix form as below:

$$\mathrm{X}=\mathrm{T}\,\mathrm{X}', \quad (6)$$

$\mathrm{X}'=\begin{bmatrix}x^{(k^{n-1})}&x^{(k^{n-2})}&\dots&x^{(1)}&x^{(0)}\end{bmatrix}$ is the $(n+1)\times 1$ column vector formed from the elements of the truthtable column vector **X** whose indexes are $0,k^{(0)},k^{(1)},k^{(n-2)},k^{(n-1)}$.

For example. for a MVL-function when k=3 and n=2 condition of linearity (5) and its matrix analogue (6) are written as below:

$$\begin{cases}x^{(0)}=x^{(0)},\\x^{(1)}=x^{(1)}\\x^{(2)}=2x^{(1)}-x^{(0)}\\x^{(3)}=x^{(3)}\\x^{(4)}=x^{(3)}+x^{(1)}-x^{(0)}\\x^{(5)}=2x^{(1)}+x^{(3)}-2x^{(0)}\\x^{(6)}=2x^{(3)}-x^{(0)}\\x^{(7)}=x^{(1)}+2x^{(3)}-2x^{(0)}\\x^{(8)}=2x^{(1)}+2x^{(3)}-3x^{(0)}\end{cases}\begin{bmatrix}x^{(0)}\\x^{(1)}\\x^{(2)}\\x^{(3)}\\x^{(4)}\\x^{(5)}\\x^{(6)}\\x^{(7)}\\x^{(8)}\end{bmatrix}=\begin{bmatrix}0&0&1\\0&1&0\\0&2&-1\\1&0&0\\1&1&-1\\1&2&-2\\2&0&-1\\2&1&-2\\2&2&-3\end{bmatrix}\begin{bmatrix}x^{(3)}\\x^{(1)}\\x^{(0)}\end{bmatrix}.$$

4.CONDITION TO REPRESENT A SYSTEM OF SOME MVL-FUNCTIONS BY A LINEAR AP

Write the condition. (5) for the system of MVL-functions given by vector X_d (1). A system of m k-valued logic functions $f_0(X), f_1(X),\dots,f_{m-1}(X)$ of n variables may be repsented by a linear AP D(X) if and only if, i-th element ($i = \overline{0, k^n - 1}$) of the truthtable column vector $\mathbf{X}_d$ of this system meets the condition

$$d^{(i)} = \sum_{j=0}^{n-1} d^{(k^j)} i_{n-j} + d^{(0)}(1 - \sum_{j=1}^{n} i_j). \tag{7}$$

The condition in matrix form is written as

$\mathbf{X}_d = \mathbf{T}\mathbf{X}'_d$,

where $$\mathrm{X}'_d = \begin{bmatrix} d^{(k^{n-1})} & d^{(k^{n-2})} & \dots & d^{(1)} & d^{(0)} \end{bmatrix} \tag{8}$$

is the column vector formed from the elements of the truthtable column vector $\mathbf{X}_d$ whose indexes are $0, k^{(0)}, k^{(1)}, k^{(n-2)}, k^{(n-1)}$. Conditions (5)-(8) are used to find whether a system of MVL-functions is represented by a linear AP.

For example find if the system of the functions $f_0(X) = x_2, f_1(X) = x_1$ $(k = 3, n = 2)$ is presented by a linear AP.

The truthtable column vectors of the initial functions. These are X_0 = [0 1 2 0 1 2 0 1 2] and X_1 =[0 0 0 1 1 1 2 2 2] and according to (1) the vector X_d is equal $\mathrm{X}_d = \mathrm{R}^0 \mathrm{X}_0 + \mathrm{R}^1 \mathrm{X}_1$ = [0 1 2 3 4 5 6 7 8]. Since X'_d = [3 1 0] then condition (8) is truth, and the assumed system of MVL-functions can be represented by the linear AP computed by

$$\mathrm{P} = K_2 \mathrm{X}_d = \begin{bmatrix} 0 & 1 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; P(X) = x_1 + 3x_2. (3)$$

The condition obtained in this section allows to bring out an opportunity to represent a system of MVL-functions by an AP. The cardinality of a class of the functions met condition (5) is not great and equal (k+2n) functions, when the number of MVL-functions is k^{k^n}. Some limits of exploitation of linear AP to represent MVL-functions are due to this fact.

5.REPRESENTATION OF SOME MVL-FUNCTIONS BY A SYSTEM OF LINEAR AP

If the conditions of existance of a linear AP are not fulfilled then the following approach is proposed.

Represent the truthtable column vector $\mathbf{X}$ of an initial MVL-function f(X) by the system of m column vectors $\mathbf{X}_0, \mathbf{X}_1, \dots, \mathbf{X}_{m-1}$, where $m =]k^n / (n+1)[$.]a[is the least integer proximated to a. In case of need the vector X is supplemented with zeros to the required size. Write the orthogonal transform in the basis K_{n+1} over each vector $\mathbf{X}_j$ ($j = \overline{0, m-1}$) of the system

$$\mathbf{P}_j = K_{n+1} \mathbf{X}_j, \tag{9}$$

where the matrix K_{n+1} is found from the matrix equation $K_{n+1} K_{n+1}^{-1} = I_{n+1}; I_{n+1}$ is the $(n+1) \times (n+1)$ identity matrix; the (p,s)-th element $k_{p,s}^{-1}$ of the matrix K_{n-1}^{-1} is formed by the rule

$$k_{p,s}^{-1} = l_1^{q_1} l_2^{q_2} \dots l_n^{q_n}; \quad (l, q = 0, k^0, k^1, \dots, k^{n-1}), \tag{10}$$

l_j and q_j are j-th digits of the k-valued representation of l and q ($j = \overline{1, n}$). Several matrices K_{n+1}^{-1} and K_{n+1} are given in Table 3.

Table 3. Direct K_{n+1} and inverse K_{n+1}^{-1} transform matrices

n	K_{n+1}	K_{n+1}^{-1}
2	$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
3	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$
4	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

Example Represent the MVL-function $f(X) = x_1 \vee x_2$ (k=5, n=2) given by its truthtable column vector X= [0 1 2 3 4 1 1 2 3 4 2 2 2 3 4 3 3 3 3 4 4 4 4 4 4] by a linear AP. Divide the vector X into $m =]k^n / (n+1)[= 9$ vectors

$$\mathbf{X}_0 = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}, \mathbf{X}_1 = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix}, \mathbf{X}_2 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix},$$
$$\mathbf{X}_3 = \begin{bmatrix} 4 & 2 & 2 \end{bmatrix}, \mathbf{X}_4 = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}, \mathbf{X}_5 = \begin{bmatrix} 3 & 3 & 3 \end{bmatrix},$$
$$\mathbf{X}_6 = \begin{bmatrix} 3 & 4 & 4 \end{bmatrix}, \mathbf{X}_7 = \begin{bmatrix} 4 & 4 & 4 \end{bmatrix}, \mathbf{X}_8 = \begin{bmatrix} 4 & 0 & 0 \end{bmatrix},$$

supplementing last vector with zeroes. Then execute orthogonal transform (9) over each from them

$$\mathrm{P}_0 = K_3 \mathrm{X}_0 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}; \mathrm{P}_1 = \begin{bmatrix} 3 & 1 & -2 \end{bmatrix}$$

Analogousy receive:

$$\mathrm{P}_2 = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}, \mathrm{P}_3 = \begin{bmatrix} 4 & -2 & -2 \end{bmatrix}; \mathrm{P}_4 = \begin{bmatrix} 2 & 1 & 2 \end{bmatrix},$$
$$\mathrm{P}_5 = \begin{bmatrix} 3 & 0 & 0 \end{bmatrix}, \mathrm{P}_6 = \begin{bmatrix} 3 & 1 & 1 \end{bmatrix}; \mathbf{P}_7 = \begin{bmatrix} 4 & 0 & 0 \end{bmatrix}; \mathrm{P}_8 = \begin{bmatrix} 4 & -4 & -4 \end{bmatrix}$$

Form the system of linear AP $P_0(X) = x_2 + 2x_1; P_1(X) = 3 + x_2 - 2x_1; \dots P_8(X) = 4 - 4x_2 - 4x$

The computation of the elements of the vector X executes by of substitution of the basic sets of values of variables x_1 and x_2 (00, 01, 10) into each polynomial of the system (Table 4).

6.COMPUTING THE COEFFICIENTS OF AP ON LINEAR SYSTOLIC ARRAYS

The array consists (n-1) CCs, every CC includes a switch (S), accumulator(A), block of FIFO registers ($Rg_1, Rg_2, \dots, Rg_m$), generator of basic sets of variables (GN) and rate control block (CB). Arithmetic sign of data, coming from Rg_1 to CB is changed when synthesizing linear AP and isn't changed when computing. The functioning of the systolic array when synthezing linear AP is realized as follows. Previously the elements of vectors $\mathbf{X}_0, \mathbf{X}_1, \dots, \mathbf{X}_{m-1}$ are written in to block of registers of all CCs, such that the elements of vector $\mathbf{X}_0$ are written into the first registers of all CC.the elements of $\mathbf{X}_1$ are written into the second registers and etc.Values of variables $x_1 = 0, x_2 = 0, \dots, x_n = 0$ of basic set from GN become to the control inputs of all swithes during the first cycle. This cycle is repeated (m-1) times, and the coefficients of the full system of linear AP, describing the initial vector X, are formed. The solution of the inverse problem (computing the values of vector X) is analogously fulfilled, but elements of vectors $\mathbf{P}_j$ are written in block of register. Thus to compute the elements of the vectors $\mathbf{P}_j$ or $\mathbf{X}$ it is required k^n cycles of the array's functioning

Table 4.Representation of MVL-function $f(X) = x_1 \vee x_2$ by a system of linear arithmetical polynomials (k=5, n=2)

Linear AP	Variables				Sub-vectors of vector **X**		Vector **X**
	Basic		Ordered				
	x_1	x_2	x_1	x_2			
$P_0(X) = x_2 + 2x_1$	0	0	0	0	$\mathbf{X}_0$	0	0
	0	1	0	1		1	1
	1	0	0	2		2	2
$P_1(X) = 3 + x_2 - 2x_1$	0	0	0	3	$\mathbf{X}_1$	3	3
	0	1	0	4		4	4
	1	0	1	0		1	1
$P_2(X) = 1 + x_2 + 2x_1$	0	0	1	1	$\mathbf{X}_2$	1	1
	0	1	1	2		2	2
	1	0	1	3		3	3
$P_3(X) = 4 - 2x_2 - 2x_1$	0	0	1	4	$\mathbf{X}_3$	4	4
	0	1	2	0		2	2
	1	0	2	1		2	2
$P_4(X) = 2 + x_2 + 2x_1$	0	0	2	2	$\mathbf{X}_4$	2	2
	0	1	2	3		3	3
	1	0	2	4		4	4
$P_5(X) = 3$	0	0	3	0	$\mathbf{X}_5$	3	3
	0	1	3	1		3	3
	1	0	3	2		3	3
$P_6(X) = 3 + x_2 + x_1$	0	0	3	3	$\mathbf{X}_6$	3	3
	0	1	3	4		4	4
	1	0	4	0		4	4
$P_7(X) = 4$	0	0	4	1	$\mathbf{X}_7$	4	4
	0	1	4	2		4	4
	1	0	4	3		4	4
$P_8(X) = 4 - 4x_2 - 4x_1$	0	0	4	4	$\mathbf{X}_8$	4	4
	0	1	-	-		0	-
	1	0	-	-		0	-

8.CONCLUSION

The linear arithmetical polynomyals "proximated" an initial MVL-function have a number of featuters being important for to organize computations: each of them describes only $m =]k^n/(n+1)[$-th part of initial funcition; the values of polynomials are computed for the same sets of variables, i.e. basic sets (independently from sets of variables functions); the values of polynomials are computed only with help of arithmetical operations of addition and substruction (the sets of variables have values "0" or "1" that excludes multiplication); the polynomial of k-valued functions are not depend on k (only their number and values of coefficients); the polynomial are computed in integer arithmetical excludes errors; the computing and synthesis of polynomials are fulfilled by means of matrix transforms that allows to receive parallel algorithms; the polynomials of the system are not depended on each from other that allows to form them and compute their values in parallel.

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